Dark Matter and Cosmic Strings in Particle Models*

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Abstract

In this talk, I will discuss the mechanism of nonthermal production of the neutralino cold dark matter from the decay of cosmic strings and the embedded defects, the π and η' strings in the strong interaction sector of the standard model.

1 Non-thermal Production of Neutralino Cold Dark Matter from Cosmic String Decays

To begin with, we consider a general case and calculate the relic mass density of the lightest supersymmetric particle(LSP)[1], then we will move on to a discussion of some implications. Consider a phase transition which is induced by the vacuum expectation value (vev) of some Higgs field Φ, $\langle|\Phi|\rangle = \eta$, and takes place at a temperature T_c with $T_c \simeq \eta$. The strings are formed by the Higgs field Φ and gauge field A. The mass per unit length of the strings is given by $\mu = \eta^2$. During the phase transition, a network of strings forms, consisting of both infinite strings and cosmic string loops. After the transition, the infinite string network coarsens and more loops form from the

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intercommuting of infinite strings. Cosmic string loops loose their energy by emitting gravitational radiation. When the radius of a loop becomes of the order of the string width, the loop releases its final energy into a burst of Φ and A particles. Those particles subsequently decay into LSP, which we denote by χ, with branching ratios ϵ and ϵ'. For simplicity we now assume that all the final string energy goes into Φ particles. A single decaying cosmic string loop thus releases $N \simeq 2\pi\lambda^{-1}\epsilon$ LSPs which we take to have a monochromatic distribution with energy $E \sim \frac{T^c}{2}$ and λ is the Higgs self coupling constant.

In such scenarios, we thus have two sources of cold dark matter(CDM) which will contribute to the matter density of the universe. We have CDM which comes from the standard scenario of thermal production; it gives a contribution to the matter density Ω_{therm}. And we also have non-thermal production of CDM which comes from the decay of cosmic string loops and gives a contribution Ω_{nonth}. The total CDM density is $\Omega_{CDM} = \Omega_{therm} + \Omega_{nonth}$. During the temperature interval between T_c and the LSP freezeout temperature T_χ, LSPs released by decaying comic string loops will thermalise very quickly with the surrounding plasma, and hence will contribute to Ω_{therm}, which should not sensitively deviate from the value calculated by the standard method. However, below the LSP freezeout temperature, since the annihilation of the LSP is by definition negligible, each CDM particle released by cosmic string decays will contribute to Ω_{nonth}. We obviously must have

$$\Omega_{nonth} < 1. \tag{1}$$

This will lead us to a constraint (a lower bound) on the cosmic string forming scale. We now calculate Ω_{nonth}.

We assume that the strings evolve in the friction dominated regime so that the very small scale structure on the strings has not formed yet. The network of strings can then be described by a single length scale $\xi(t)$. In the friction dominated period, the length scale $\xi(t)$ has been shown to scale as [2]:

$$\xi(t) = \xi(t_c)\left(\frac{t}{t_c}\right)^{\frac{3}{2}} \tag{2}$$

where $\xi(t_c) \sim (\lambda\eta)^{-1}$. The number density of cosmic string loops created per

unit of time is given by [3]:

$$\frac{dn}{dt} = \nu \xi^{-4} \frac{d\xi}{dt} \tag{3}$$

where ν is a constant of order 1. We are interested in loops decaying below T_χ. The number density of LSP released from t_{lsp} till today is given by:

$$n_{lsp}^{nonth}(t_0) = N\nu \int_{\xi_F}^{\xi_0} \left(\frac{t}{t_0}\right)^{\frac{3}{2}} \xi^{-4} d\xi \tag{4}$$

where the subscript 0 refers to parameters which are evaluated today. $\xi_F = \xi(t_F)$ where t_F is the time at which cosmic string loops which are decaying at time t_χ (associated with the LSP freezeout temperature T_χ) have formed. Now the loop's average radius shrinks at a rate [3] $\frac{dR}{dt} = -\Gamma_{loops} G\mu$, where Γ_{loops} is a numerical factor $\sim 10 - 20$. Since loops form at time t_F with an average radius $R(t_F) \simeq \lambda^{-1} G\mu M_{pl}^{\frac{1}{2}} t_F^{\frac{3}{2}}$, they have shrunk to a point at the time $t \simeq \lambda^{-1} \Gamma_{loops}^{-1} M_{pl}^{\frac{1}{2}} t_F^{\frac{3}{2}}$. Thus $t_F \sim (\lambda\Gamma)_{loops}^{\frac{2}{3}} M_{pl}^{-\frac{1}{3}} t_\chi^{\frac{2}{3}}$. Now the entropy density is $s = \frac{2\pi^2}{45} g_* T^3$ where g_* counts the number of massless degrees of freedom in the corresponding phase. The time t and temperature T are related by $t = 0.3 g_*^{-\frac{1}{2}}(T) \frac{M_{pl}}{T^2}$ where M_{pl} is the Planck mass. Thus using Eqs.(2) and (4), we find that the LSP number density today released by decaying cosmic string loops is given by:

$$Y_{LSP}^{nonth} = \frac{n_{lsp}^{nonth}}{s} = \frac{6.75}{\pi} \epsilon \nu \lambda^2 \Gamma_{loops}^{-2} g_{*T_c}^{\frac{-9}{4}} g_{*T_\chi}^{\frac{3}{4}} \, M_{pl}^2 \, \frac{T_\chi^4}{T_c^6} \, , \tag{5}$$

where the subscript on g^* refers to the time when g^* is evaluated.

The LSP relic abundance is related to Y_χ by:

$$\begin{aligned} \Omega_\chi h^2 &\approx M_\chi Y_\chi s(t_0) \rho_c(t_0)^{-1} h^2 \\ &\approx 2.82 \times 10^8 \, Y_\chi^{tot} \, (M_\chi/\mathrm{GeV}) \end{aligned} \tag{6}$$

where h is the Hubble parameter and M_χ is the LSP mass. Now $Y_{LSP}^{tot} = Y_\chi^{therm} + Y_\chi^{nonth}$; hence by setting $h = 0.70$, Eqs. (6) and (1) lead to the following constraint:

$$5.75 \times 10^8 \, Y_\chi^{nonth} \, (M_\chi/\mathrm{GeV}) < 1. \tag{7}$$

We thus see that Eqs. (5) and (7) lead to a lower bound on the cosmic string forming temperature T_c.

Our results have important implications for supersymmetric extensions of the standard model with extra $U(1)$'s (or grand unified models with an intermediate $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$ gauge symmetry). Most importantly, the requirement $\Omega_{nonth} < 1$ imposes a new constraint on supersymmetric model building and rules out many models with a low scale of a new symmetry breaking which produces defects such as cosmic strings.

Consider, for example, the model with an extra $U_{B-L}(1)$ gauge symmetry. In this model, the strings will release not only right-handed neutrinos N_i, but also their superpartners $\tilde{N}_i$. The heavy neutrinos N_i and their scalar partners $\tilde{N}_i$ can decay into various final states including the LSP. The superpotential relevant to the decays is

$$W = H_1 \epsilon L y_l E^c + H_2 \epsilon L y_\nu N^c,$$

where H_1, H_2, L, E^c and N^c are the chiral superfields and y_l, y_ν are Yukawa couplings for the lepton and neutrino Dirac masses, $m_l = y_l v_1, m_D = y_\nu v_2$, with $v_{1,2}$ being the vacuum expectation values of the Higgs fields. At tree level, the decay rates of N_i into s-lepton plus Higgsino and lepton plus Higgs are the same and they are smaller than the rate of $\tilde{N}_i$ decaying into s-lepton plus Higgs and Higssino plus lepton by a factor of 2. If the neutralino is higgsino-like, the LSP arise directly from the decays of the N_i and $\tilde{N}_i$. If the neutralino is bino- or photino-like, subsequent decays of s-lepton into binos or photinos plus leptons will produce the LSP. For reasonable values of the parameters, we estimate the branching ratio ϵ of the heavy particle decay into LSP to be between 0.1 and 0.5. From Eq. (5) it follows that string decays can easily produce the required amount of LSP. However too many LSPs will be generated unless the $B-L$ breaking scale, Λ_{B-L} is higher than about 10^8 GeV . In turn, this will set a lower limit on the neutrino masses generated by the see-saw mechanism, $m_\nu \sim m_D^2/\Lambda_{B-L}$. Inserting numbers and taking $m_D \sim m_\tau \sim 1.8$ GeV, one obtains that $m_\nu \leq 30$ eV.

Our lower limit on the $B-L$ symmetry breaking scale in gauged $B-L$ models and in general models with an extra $U(1)$ [4] pushes the mass of the new gauge boson far above the Fermi scale, rendering it impossible to test the new physics signals from the extra Z' in accelerators.

As one more implication of our results, we consider the limit on the lifetime of the Z-string[5] in the MSSM. Since the Z-string is produced during

the electroweak phase transition, the stable Z-string would produce too much LSP and overclose the universe. However if the Z-string decays before the temperature of the LSP freezeout, then the LSP produced from the decay of the Z-string would be thermalized immediately, which would result in a negligible Ω_{nonth}.

2 π and η' strings in QCD

We consider an idealization of QCD with two species of massless quarks u and d. The lagrangian of strong interaction physics is invariant under $SU_L(2) \times SU_R(2)$ (we will come back to the discussion of the $U_A(1)$ at end of this section) chiral transformations

$$\Psi_{L,R} \to \exp(-i\vec{\theta}_{L,R} \cdot \vec{\tau})\Psi_{L,R}, \tag{8}$$

where $\Psi^T_{L,R} = (u,d)_{L,R}$. However this chiral symmetry does not appear in the low energy particle spectrum since it is spontaneously broken due to the quark condensate. Consequently, three Goldstone bosons, the pions, appear and the (constituent) quarks become massive. At low energy, the spontaneous breaking of chiral symmetry can be described by an effective theory, the linear sigma model, which involves the massless pions $\vec{\pi}$ and a massive σ particle.

As usual, we introduce the field

$$\Phi = \sigma\frac{\tau^0}{2} + i\vec{\pi}\frac{\vec{\tau}}{2}, \tag{9}$$

where τ^0 is unity matrix and $\vec{\tau}$ are the Pauli matrices with the normalization condition $Tr(\tau^a\tau^b) = 2\delta^{ab}$. Under $SU_L(2) \times SU_R(2)$ chiral transformations, Φ transforms as

$$\Phi \to L^+\Phi R. \tag{10}$$

The renormalizable effective lagrangian of the linear sigma model is given by

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_q, \tag{11}$$

where

$$\mathcal{L}_\Phi = Tr[(\partial_\mu\Phi)^+\partial^\mu\Phi] - \lambda[Tr(\Phi^+\Phi) - \frac{f_\pi^2}{2}]^2, \tag{12}$$

and

$$\mathcal{L}_q = \bar{\Psi}_L i\gamma^\mu \partial_\mu \Psi_L + \bar{\Psi}_R i\gamma^\mu \partial_\mu \Psi_R - 2g\bar{\Psi}_L \Phi \Psi_R + h.c.. \tag{13}$$

During chiral symmetry breaking, the field σ takes on a nonvanishing vacuum expectation value, which breaks $SU_L(2) \times SU_R(2)$ down to $SU_{L+R}(2)$. This results in a massive sigma σ and three massless Goldstone bosons $\vec{\pi}$, as well as giving a mass $m_q = gf_\pi$ to the constituent quarks. Numerically, $f_\pi \sim 94$ MeV and $m_q \sim 300$ MeV.

We studied the classical solutions of this model and discovered a class of vortex-like configurations which we refer to as pion string[6]. Like the Z string[5] of the standard electroweak model, the pion string is not topologically stable. Nevertheless, as demonstrated recently in numerical simulations in the case of semilocal strings[7] (which are also not topologically stable) pion strings are expected to be produced during the QCD phase transition in the early Universe (and also in heavy-ion collisions). The strings will subsequently decay.

The pion string is a static configuration of the lagrangian $\mathcal{L}_\Phi$ of Eq. (12). To construct these solutions, we define new fields

$$\phi = \frac{\sigma + i\pi^0}{\sqrt{2}}, \tag{14}$$

$$\pi^\pm = \frac{\pi^1 \pm i\pi^2}{\sqrt{2}}. \tag{15}$$

The lagrangian $\mathcal{L}_\Phi$ now can be rewritten as

$$\mathcal{L} = (\partial_\mu \phi)^* \partial^\mu \phi + \partial_\mu \pi^+ \partial^\mu \pi^- - \lambda(\pi^+\pi^- + \phi^*\phi - \frac{f_\pi^2}{2})^2. \tag{16}$$

For static configurations, the energy functional corresponding to the above lagrangian is given by

$$E = \int d^3x \left[\vec{\bigtriangledown}\phi^* \vec{\bigtriangledown}\phi + \vec{\bigtriangledown}\pi^+ \vec{\bigtriangledown}\pi^- + \lambda(\pi^+\pi^- + \phi^*\phi - \frac{f_\pi^2}{2})^2 \right]. \tag{17}$$

The time independent equations of motion are:

$$\bigtriangledown^2 \phi = 2\lambda(\pi^+\pi^- + \phi^*\phi - \frac{f_\pi^2}{2})\phi, \tag{18}$$

$$\bigtriangledown^2 \pi^+ = 2\lambda(\pi^+\pi^- + \phi^*\phi - \frac{f_\pi^2}{2})\pi^+. \tag{19}$$

The pion string solution extremising the energy functional of Eq. (17) is given by

$$\phi = \frac{f_\pi}{\sqrt{2}}\rho(r)e^{i\theta}, \qquad (20)$$

$$\pi^{\pm} = 0, \qquad (21)$$

where the coordinates r and θ are polar coordinates in the $x-y$ plane (the string is assumed to lie along the z axis), and $\rho(r)$ satisfies the following boundary conditions

$$r \to 0, \qquad \rho(r) \to 0; \qquad (22)$$

$$r \to \infty, \qquad \rho(r) \to 1. \qquad (23)$$

The pion-string is not topologically stable, since any field configuration can be continuously deformed to the vacuum.

In the discussions above we have neglected the electromagnetic interaction. When turning on $U_{em}(1)$, the derivatives are replaced by the covariant derivatives. Recently Nagasawa and Brandenberger[8] have pointed out that the interactions with a finite temperature plasma will lead to corrections to the effective potential and may stabilize the π strings.

Since a pion string is made of σ and π^0 fields, it is neutral under the $U_{em}(1)$ symmetry. However, the π^0 will couple to photons via the Wess Zumino type interaction.

$$\mathcal{L}_{low} = \frac{f_\pi^2}{4}Tr(\partial_\mu \Sigma^+ \partial^\mu \Sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{N_c \alpha}{24\pi}\frac{\pi^0}{f_\pi}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}, \qquad (24)$$

where $N_c = 3$, $\Sigma = exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi)$, and α is the electromagnetic fine structure constant.

Recently Brandenberger and I[9] propose a new mechanism for the generation of primordial magnetic fields. It is based on the realization that anomalous global strings couple to electricity and magnetism via the anomalous interactions shown above and induce magnetic fields . The major advantage of this mechanism is that the coherence scale of the magnetic fields induced by these global vortex lines is set by the length scale $\xi(t)$ of the strings (the typical curvature radius of the strings).

Before concluding this section, I will speculate about the existence of an unstable η' string[6]. In QCD, in the limit of massless quarks, there is an

additional $U_A(1)$ chiral symmetry. This chiral symmetry, when broken by the quark condensate, predicts the existence of a goldstone boson. There is no such a light meson, however. This is resolved by the Adler-Bell-Jackiw $U_A(1)$ anomaly together with the properties of non-trivial vacuum structure of non-abelian gauge theory, QCD. The $U_A(1)$ symmetry is badly broken by instanton effects at zero temperature.

As the density of matter and/or the temperature increases, it is expected that the instanton effects will rapidly disappear, and one thus has an additional $U_A(1)$ symmetry (besides $SU_L(2) \times SU_R(2)$) at the transition temperature of the QCD chiral symmetry. When the $U_A(1)$ symmetry is broken spontaneously by the quark condensate, a topological string, the η'-string, results. Differing from the pion-string, the η'-string is topologically stable at high temperatures, but will decay as the temperature decreases. The η'-string can form during the chiral phase transition of QCD. In the setting of cosmology, it will exist during a specific epoch below the QCD chiral symmetry breaking temperature during the evolution of the universe. In the context of heavy-ion collisions, it will exist in the plasma created by the collision. The strings then become unstable as the temperature decreases and when the instanton effects become substantial.

Acknowledgments

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References

[1] R. Jeannerot, X. Zhang and R. Brandenberger, JHEP 9912 (1999) 003.

[2] R. Brandenberger and A. Riotto, Phys. Lett. B445 (1999) 323.

[3] A. Vilenkin and E.P.S. Shellard, *"Cosmic strings and other topological defects"* (Cambridge University Press, Cambridge, 1994); M. Hindmarsh and T.W.B. Kibble, *Rept. Prog. Phys.* **58**, 477 (1995); R. Brandenberger, *Int. J. Mod. Phys.* **A9**, 2117 (1994).

[4] P. Langacker, "Z-Prime physics from strings", hep-ph/9805486.

[5] T. Vachaspati, Phys. Rev. Lett. 68, (1992) 1977.

[6] X. Zhang, T. Huang and R. Brandenberger, Phys. Rev. D58, (1998) 027702.

[7] A. Achucarro, J. Borrill and A. Liddle, hep-ph/9802306.

[8] M. Nagasawa and R. Brandenberger, Phys. Lett. B467 (1999) 205.

[9] R. Brandenberger and X. Zhang, Phys. Rev. D59 (1999) 081301.